Examen 3 (solutions)

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Q.1. Points d'intersection : $x^3 = 4x^2 - 3x \Rightarrow x^3 - 4x^2 + 3x = x(x-1)(x-3) =$ donc $x = \{0, 1, 3\}$. L'aire est :

$$\int_{-1}^{0} -x^3+4x^2-3x\,dx+\int_{0}^{1} x^3-4x^2+3x\,dx+\int_{1}^{3} -x^3+4x^2-3x\,dx+\int_{3}^{4} x^3-4x^2+3x\,dx$$

$$= -\frac{x^4}{4}+\frac{4x^3}{3}-\frac{3x^2}{2}\Big|_{-1}^{0}+\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}\Big|_{0}^{1}+-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{3x^2}{2}\Big|_{1}^{3}+\frac{x^4}{4}-\frac{4x^3}{3}+\frac{3x^2}{2}\Big|_{3}^{4}$$

$$= \frac{4^4-2(3)^3+3}{4}+\frac{8(3)^3-4(4)^3-4}{3}+\frac{3(4)^2-6(3)^2+9}{2} = \frac{205}{4}-\frac{44}{3}+\frac{3}{2} = \frac{457}{12}$$

Q.2. a) $\pi\int_{-1}^{2}(x^2+1)^2\,dx = \pi\int_{-1}^{2} x^4+2x^2+1\,dx = \pi\left(\frac{x^5}{5}+\frac{2x^3}{3}+x\right)\Big|_{-1}^{2}$

$\pi\left(\frac{2^5}{5}+\frac{2^4}{3}+2\right)-\pi\left(-\frac{1}{5}-\frac{2}{3}-1\right) = \frac{78\pi}{5}$

b) $\pi\int_{0}^{\frac{\pi}{4}}\left(\frac{2}{\sqrt{2}}\right)^2\,dy-\pi\int_{0}^{\frac{\pi}{4}}\sec^2 y\,dy = 2\pi y\Big|_{0}^{\frac{\pi}{4}}-\pi\tan y\Big|_{0}^{\frac{\pi}{4}} = \frac{\pi^2}{2}-\pi$

c) $2\pi\int_{0}^{\frac{1}{2}}(x+2)(3-x^2-e^x)dx = 2\pi\int_{0}^{\frac{1}{2}} -xe^x-e^x-x^3-2x^2+3x+6\,dx$

$= 2\pi\left(-xe^x+\cancel{e^x}-\cancel{e^x}-\frac{x^4}{4}-\frac{2x^3}{3}+\frac{3x^2}{2}+6x\right)\Big|_{0}^{\frac{1}{2}} = 2\pi\left(\frac{629}{192}-\frac{e^{\frac{1}{2}}}{2}\right)$

Q.3. $\int_{-3}^{1}\sqrt{1+\left(\frac{e^x-e^{-x}}{2}\right)^2}dx = \int_{-3}^{1}\sqrt{1+\frac{e^{2x}}{4}-\frac{1}{2}+\frac{e^{-2x}}{4}}dx = \int_{-3}^{1}\sqrt{\frac{e^{2x}}{4}+\frac{1}{2}+\frac{e^{-2x}}{4}}dx$

$\int_{-3}^{1}\sqrt{\left(\frac{e^x+e^{-x}}{2}\right)^2}dx = \frac{1}{2}(e^x-e^{-x})\Big|_{-3}^{1} = \frac{1}{2}\left(e-\frac{1}{e}-\frac{1}{e^3}+e^3\right)$

Q.4. a) $\int_{-2}^{1}\frac{4x+6}{(x^2+1)(x+2)}\,dx = \lim_{a\to -2^+}\int_{a}^{1}\frac{\frac{2}{5}x+\frac{32}{10}}{x^2+1}+\frac{-2}{5(x+2)}dx$

$\lim_{a\to -2^+}\left(\frac{1}{5}\ln|x^2+1|+\frac{32}{10}\arctan x-\frac{2}{5}\ln|x+2|\right)\Big|_{a}^{1} = -\infty$ (impropre, diverge)

b) $\int_{0}^{\frac{\pi}{2}}\tan x\,dx = \lim_{a\to\frac{\pi}{2}^-}\int_{0}^{a}\tan x\,dx = \lim_{a\to\frac{\pi}{2}^-}(-\ln|\cos x|)\Big|_{0}^{a} = \infty$ (impropre, diverge)

c) $\int_{-\infty}^{\infty}\frac{dx}{\left(\frac{1}{2}+x^2\right)} = \lim_{N\to-\infty}\frac{1}{2}\int_{N}^{0}\frac{dx}{1+(\sqrt{2}x)^2}+\lim_{M\to\infty}\frac{1}{2}\int_{0}^{M}\frac{dx}{1+(\sqrt{2}x)^2}$

$= \lim_{N\to-\infty}\frac{1}{2}\arctan(\sqrt{2}x)\Big|_{N}^{0}+\lim_{M\to\infty}\frac{1}{2}\arctan(\sqrt{2}x)\Big|_{0}^{M} = \frac{\pi}{2}$ (impropre, converge)