Examen 2 (solutions)(pas fini!)

201-NYB Calcul Intégral

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Question 1.

a) $$\int xe^x \, dx = xe^x - \int e^x \, dx = xe^x - e^x + C$$

$$u = x \qquad dv = e^x dx$$
$$du = dx \qquad v = e^x$$

b) $$\int \frac{x+3}{(x-1)(x+5)} \, dx$$
$$= \int \frac{2}{3(x-1)} + \frac{1}{3(x+5)} \, dx$$
$$\frac{2\ln|x-1|}{3} + \frac{\ln|x+5|}{3} + C$$

$$\frac{A}{x-1} + \frac{B}{x+5} = \frac{A(x+5) + B(x-1)}{(x-1)(x+5)} = \frac{x+3}{(x-1)(x+5)}$$
$$x = 1 \Rightarrow A = \frac{2}{3} \text{ et } \quad x = -5 \Rightarrow B = \frac{1}{3}$$

c) $$\int \csc^5 x \cot^3 x \, dx = \int \csc^4 x \cot^2 x \csc x \cot x \, dx$$
$$= \int \csc^4 x (\csc^2 x - 1) \csc x \cot x \, dx = -\int u^4(u^2-1) \, du$$
$$-\frac{u^7}{7} + \frac{u^5}{5} + C = -\frac{\csc^7 x}{7} + \frac{\csc^5 x}{5} + C$$

$$u = \csc x$$
$$du = -\csc x \cot x dx$$

d) $$I = \int e^{5x} \sin x \, dx = -e^{5x}\cos x + 5\int e^{5x} \cos x \, dx$$
$$= -e^{5x}\cos x + 5\left(e^{5x}\sin x - 5\int e^{5x}\sin x \, dx\right)$$
$$= -e^{5x}\cos x + 5e^{5x}\sin x - 25I$$
$$\Rightarrow I = \frac{-e^{5x}\cos x + 5e^{5x}\sin x}{26} + C$$

$$u = e^{5x} \qquad dv = \sin x dx$$
$$du = 5e^{5x}dx \qquad v = -\cos x$$

$$u = e^{5x} \qquad dv = \cos x dx$$
$$du = 5e^{5x}dx \qquad v = \sin x$$

e)

$$\int \frac{x^3}{\sqrt{4-2x^2}}\ dx = \int \frac{2^4 \sin^3\theta \cos\theta d\theta}{2^3 \cos\theta}$$

$$= 2\int \sin^3\theta d\theta = 2\int (1-\cos^2\theta)\sin\theta d\theta = -2\int (1-u^2)du$$

$$= -2\cos\theta + \frac{2\cos^3}{3} + C = -\sqrt{4-2x^2} + \frac{\sqrt{(4-2x^2)^3}}{3\cdot 2^2} + C$$

$$x = \frac{2\sin\theta}{\sqrt{2}} \qquad dx = \frac{2\cos\theta}{\sqrt{2}}d\theta$$

$$\sqrt{4-2x^2} = 2\cos\theta$$

$$u = \cos\theta \quad du = -\sin\theta$$

f)

$$\int x^3 \arctan(x^2)\ dx$$

$$= \frac{1}{2}\int y\arctan(y)\ dx = \frac{1}{2}\left(\frac{y^2\arctan y}{2} - \frac{1}{2}\int \frac{y^2}{1+y^2}dy\right)$$

$$= \frac{x^4\arctan x^2}{4} - \frac{1}{4}\int 1 - \frac{1}{1+y^2}dy$$

$$= \frac{x^4\arctan x^2}{4} - \frac{x^2}{4} - \frac{\arctan x^2}{4} + C$$

$$y = x^2, dy = 2xdx$$

$$u = \arctan y \quad dv = ydy$$

$$du = \frac{dy}{1+y^2} \qquad v = \frac{y^2}{2}$$

g)

$$\int \sin^3(2x)\cos^2 x\ dx = \frac{1}{2}\int \sin^3(2x)(1+\cos(2x))\ dx$$

$$= \frac{1}{2}\int \sin^2(2x)(1+\cos(2x))\sin(2x)\ dx$$

$$= \frac{1}{2}\int (1-\cos^2(2x))(1+\cos(2x))\sin(2x)\ dx$$

$$= \frac{1}{4}\int (1-u^2)(1+u)\ du = \frac{1}{4}\int 1+u-u^2-u^3\ du$$

$$\frac{\cos(2x)}{4} + \frac{\cos^2(2x)}{8} - \frac{\cos^3(2x)}{12} - \frac{\cos^4(2x)}{16} + C$$

$$u = \cos(2x)$$

$$du = 2\sin(2x)dx$$

h)

$$\int \sqrt{e^{2x}-4}\,dx = \int \frac{\sqrt{u^2-4}}{u}du = \int \frac{2^2\tan\theta\sec\theta\tan\theta d\theta}{2\sec\theta}$$

$$= 2\int \sec^2\theta - 1 d\theta$$

$$= 2\tan\theta - 2\theta + C = \sqrt{e^{2x}-4} - 2\arctan\left(\frac{\sqrt{e^{2x}-4}}{2}\right) + C$$

$u = e^x$ $\quad du = e^x dx$

$u = 2\sec\theta$ $\quad du = 2\sec\theta\tan\theta$

$\sqrt{u^2-4} = 2\tan\theta$

i) $$\frac{x^3-2x^2+x+3}{(2+x^2)(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2+2} = \frac{A(x+1)(x^2+2)+B(x^2+2)+(Cx+D)(x+1)^2}{(2+x^2)(x+1)^2}$$

En posant $x=-1$, on a que $-1-2-1+3=-1=B(1+2)$ d'où $\boxed{B=-\frac{1}{3}}$.

$$\begin{aligned} x^3-2x^2+x+3 &= A(x^3+x^2+2x+2) - 1/3(x^2+2) + C(x^3+2x^2+x) + D(x^2+2x+1) \\ &= (A+C)x^3 + \left(A - \frac{1}{3} + 2C + D\right)x^2 + (2A+C+2D)x + \left(2A - \frac{2}{3} + D\right) \end{aligned}$$

En isolant C et D de la première et dernière égalité, on obtient $C=(1-A)$ et $D=\left(\frac{11}{3}-2A\right)$ et en remettant ça dans la troisième égalité on obtient

$2A + (1-A) + 2\left(\frac{11}{3} - 2A\right) = -3A + \frac{25}{3} = 1 \Longrightarrow \boxed{A = \frac{22}{9}} \Longrightarrow \boxed{C = -\frac{13}{9}} \Longrightarrow \boxed{D = -\frac{11}{9}}$.

$$\int \frac{x^3-2x^2+x+3}{(2+x^2)(x+1)^2}dx = \int \frac{22}{9(x+1)} - \frac{1}{3(x+1)^2} - \frac{13x+11}{(x^2+2)}dx$$

$$= \frac{22\ln|x+1|}{9} + \frac{1}{3(x+1)} - \frac{13\ln|x^2+2|}{2} - \frac{11}{\sqrt{2}}\arctan\left(\frac{x}{\sqrt{2}}\right) + C$$

j)

$$\int \frac{1+\cos x}{e^{2x}}dx = \frac{-(1+\cos x)e^{-2x}}{2} + \frac{1}{2}\int e^{-2x}\sin x dx$$

mais $I = \int e^{-2x}\sin x dx = -\frac{e^{-2x}\sin x}{2} + \frac{1}{2}\int e^{-2x}\cos x dx$

$$-\frac{e^{-2x}\sin x}{2} + \frac{1}{2}\left(-\frac{e^{-2x}\cos x}{2} - \frac{1}{2}I\right) = \dots$$

$u = 1+\cos x$ $\quad dv = e^{-2x}dx$

$du = -\sin x dx$ $\quad v = -\frac{e^{-2x}}{2}$

$u = \sin x$ $\quad dv = e^{-2x}dx$

$du = \cos x dx$ $\quad v = -\frac{e^{-2x}}{2}$

$u = \cos x$ $\quad dv = e^{-2x}dx$

$du = -\sin x dx$ $\quad v = -\frac{e^{-2x}}{2}$